

A OPTIMAL COLOR MODULATION DERIVATION

Given an ellipse constraint function

$$\begin{cases} \mathcal{E}(\mathbf{x}) = \left(\frac{x_1 - t_1}{a_1}\right)^2 + \left(\frac{x_2 - t_2}{a_2}\right)^2 - 1 = 0 \\ x_3 = t_3, \end{cases} \quad (16)$$

and a power cost function

$$\mathcal{P}(\mathbf{x}) = p_1 x_1 + p_2 x_2 + p_3 x_3 + p_{circ}, \quad (17)$$

we may solve the power optimizing \mathbf{x}^* via the method of Lagrange multipliers.

First, we notice that x_3 should not change. Intuitively, this effectively reduces the dimensionality of the optimization onto the plane $x_3 = t_3$. Formally, we may rewrite the constraint and power functions in terms of a 2-dimensional variable $\mathbf{y} = (y_1, y_2) = (x_1, x_2)$:

$$\mathcal{E}(\mathbf{y}) = \left(\frac{y_1 - t_1}{a_1}\right)^2 + \left(\frac{y_2 - t_2}{a_2}\right)^2 - 1 = 0, \quad (18)$$

and

$$\mathcal{P}(\mathbf{y}) = p_1 y_1 + p_2 y_2 + \text{const}. \quad (19)$$

The minimizing vector \mathbf{y}^* satisfies the condition that the gradients of \mathcal{E} , and \mathcal{P} are co-linear. So the system of equations we need to solve for \mathbf{y}^* is

$$\begin{cases} \nabla \mathcal{E}(\mathbf{y}^*) = \phi \nabla \mathcal{P}(\mathbf{y}^*) \\ \mathcal{E}(\mathbf{y}^*) = 0, \end{cases} \quad (20)$$

for some scalar constant ϕ .

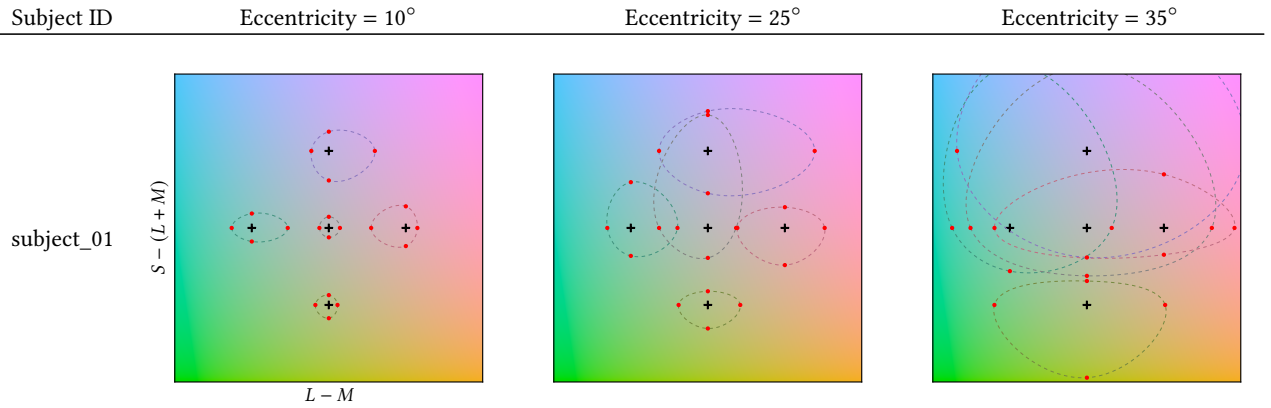
Computing the gradients, we get

$$\begin{cases} \frac{2}{a_1} \frac{y_1^* - t_1}{a_1} = \phi p_1 \\ \frac{2}{a_2} \frac{y_2^* - t_2}{a_2} = \phi p_2 \\ \left(\frac{y_1^* - t_1}{a_1}\right)^2 + \left(\frac{y_2^* - t_2}{a_2}\right)^2 - 1 = 0. \end{cases} \quad (21)$$

Finally, we solve for \mathbf{y}^* using this system of equations to get the optimal color, \mathbf{x}^* :

$$\begin{aligned} x_1^* &= \frac{p_1 a_1^2}{\sqrt{p_1^2 a_1^2 + p_2^2 a_2^2}} \\ x_2^* &= \frac{p_2 a_2^2}{\sqrt{p_1^2 a_1^2 + p_2^2 a_2^2}} \\ x_3^* &= t_3. \end{aligned} \quad (22)$$

B INDIVIDUAL PARTICIPANT DATA FOR PILOT PERCEPTUAL USER STUDY



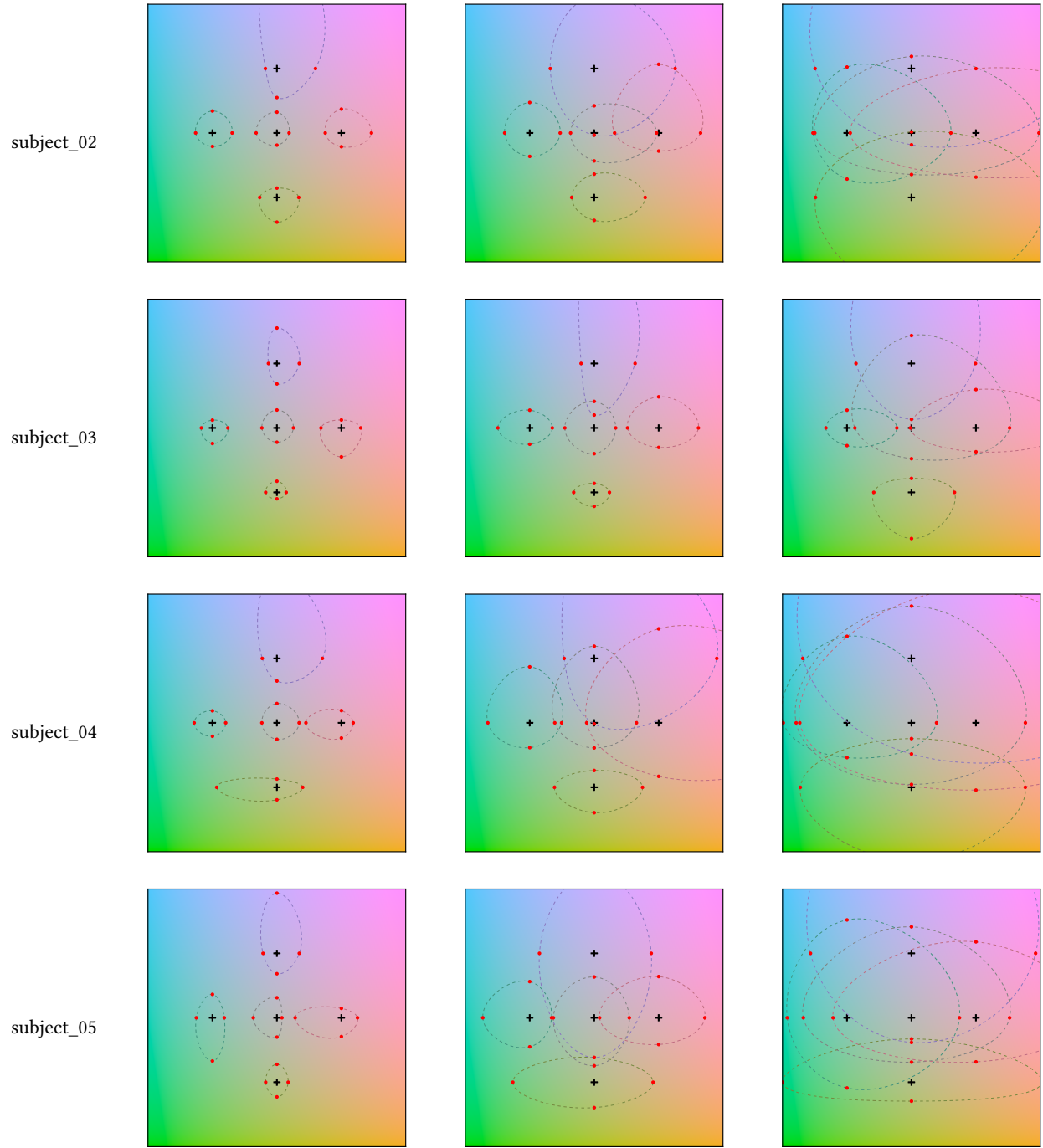


Table 3. Pilot Perceptual Study Threshold Data